**BMP selection using Analytic Network Process**

Here we outline the procedure to determine best management practices using Analytic Network Process (ANP) scheme first introduced by Saaty [cite]. ANP among different multicriteria decision making schemes allow high degree of flexibility in choosing alternatives and criteria to yield a robust decision-making framework. For our problem we modified the original ANP framework to work directly with the available alternatives instead of working with alternatives and criteria. To do this we consolidate the criteria into two, i.e., benefit (for e.g., Nitrate reduction) and cost.

In order to formulate the decision-making process and solve the same using ANP where we want to select the BMPs with the best benefits and least cost, we consider a most general case with BMPs as possible alternatives to form our ANP network nodes denoted by . Each may have corresponding criteria where denotes the number of criteria and indicates whether the criteria is related to benefit or cost . Using the above framework, we define the ANP supermatrix ; ( with corresponding weights or efficiencies of each node given by

We then populate supermatrix using the above formula as follows

.

One may note that the thus obtained supermatrix is symmetric by construction and can be normalized using either the row or column normalization scheme. Using row normalization scheme, we get , where is elementwise division and is all-ones matrix.

For the normalized supermatrix to reflect the interdependencies and feedback of the alternatives (i.e., the available BMPs) we need to stabilize

Given that the above limit is convergent and exclusive the matrix thus obtained reflects the ANP efficiencies of the corresponding BMP alternatives.

Selecting the best performing BMPs from all the available BMP set of is trivial from and can be achieved using the operation . The operation can be repeated to get next best BMP set in a similar manner.

A diagram of a network structure

Description automatically generated



Figure . The loop for alternatives indicates the inner dependence among the elements of the alternative in regard to a common property. The arc, for example, from alternative to , indicates the outer dependence among the elements in on the elements in with respect to a common property.